Grenzüberschreitungen

Studien zur Kulturgeschichte des Alten Orients

Festschrift für Hans Neumann
zum 65. Geburtstag am 9. Mai 2018

Herausgegeben von Kristin Kleber,
Georg Neumann und Susanne Paulus
unter Mitarbeit von Christin Möllenbeck

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Was Babylonian Mathematics Algorithmic?

Jens Høyrup

Preliminaries: "Algorithms", "Babylonian", "mathematics"

In the wake of the discoveries of Otto Neugebauer and François Thureau-Dangin it became customary to speak of "Babylonian algebra", and even of the generally algebraic character of Mesopotamian mathematics – not least because mathematics going beyond the most elementary level was supposed to be either geometric (in Greek style, which Mesopotamian mathematics was not) or algebraic. This, we may say, portrays the 18th-century understanding of mathematics, where d'Alembert – a master of the symbol-carried analysis that had been constructed in the previous century but only unfolded after 1700 – belonged to the class of *géomètres* because his kind of mathematics was based on proofs, and proofs belonged with geometry.

The youngest generation of mathematicians has not been brought up with Euclidean-style geometry in secondary school – at least in this respect, the new-math reform as summarized in Jean Dieudonné's slogan "à bas les triangles, à bas Euclide!" was successful.[1] Instead, it has had to discover the now classical kind of mathematics (growing out of and beyond analytic geometry and infinitesimal analysis and submitting to its rule and methods even classical geometry) to be, at least in social importance, the junior partner of computer science. Both partners of course try to build on mathematical truth; but whereas the classical kind (thought of as an ideal type – there is ample space for modifications) regards *the proof* as its essence, proofs are considered (ideally again) in computer science a requisite ascertaining the reliability of procedures, and *procedures* are aim and essence; if more or less formal proofs cannot be constructed (and for many advanced procedures they cannot), "the proof of the pudding is the eating".[2]

---

[1] Formulated during the discussion at the "Colloque de Royaumont" in 1959, quoted in Castelnuovo, E. 1977, 43. Others remember the even more colourful "mort aux triangles", still others the more balanced "plus de triangles". Challenged in the discussion, Dieudonné may well have said all of it – that was his habit.

[2] In the words of a textbook (Acton, F.S. 1990, xvii): "It is a commonplace that numerical processes that are efficient usually cannot be proven to converge, while those amenable to proof are inefficient [...]. The best demonstration of convergence is convergence itself."

Procedures that are to be implemented in a machine have to be precisely defined – they must be *algorithms* (perhaps, as in present-day artificial intelligence, algorithms for constructing new algorithms according to feed-back). As explanation of what that means, let me quote a recent basic textbook:

> Informally, an algorithm is any well-defined computational procedure that takes some value, or set of values, as input and produces some value, or set of values, as output. An algorithm is thus a sequence of computational steps that transform the input into the output.
>
> We can also view an algorithm as a tool for solving a well-specified computational problem. The statement of the problem specifies in general terms the desired input/output relationship. The algorithm describes a specific computational procedure for achieving that input/output relationship.[3]

The algorithms we all know are those for computing with "Arabic" numerals. We may look at the addition

$$\begin{array}{lr} & a(n)\ \ldots\ a(2)\ a(1) \\ & \underline{b(n)\ \ldots\ b(2)\ b(1)} \\ c(n{+}1) & c(n)\ \ldots\ c(2)\ c(1) \end{array}$$

informally, the algorithm tells us to calculate $a(1)+b(1) = t$; IF $t<10$, $c(1) = \mathrm{t}$, ELSE $c(1) = t–10$, and $a(2)$ is augmented by 1. Next we move one place to the left, and repeat the process, doing so until we have reached n; here, IF $t = a(n)+b(n)<10$, $c(n) = t$, $c(n+1) = 0$, ELSE $c(n) = t–10$, $c(n+1) = 1$. In a formal algorithm (the one required for a computer), we should start by putting $i = 1$, making the addition $a(i)+b(i)$ with ensuing branching as described, augment i by 1, and repeat the process UNTIL $i = n$.

In consequence of the disappearance from view of the distinction between the geometric and the algebraic/analytical types of mathematics and the overwhelming growth of computer science, the traditional *tertium non datur* of historians of mathematics has been replaced by another one, according to which mathematics which is not of the classical, proof-centred type must be *algorithmic*.[4]

The algorithm concept was supposedly introduced as a general historiographic tool by Donald Knuth in 1972. Knuth's actual purpose was not to interpret history but to provide computer science with cultural legitimacy, by showing that its central tool – algorithms – had a long prehistory. Thus the very first words of the article:

---

[3] Cormen, Th.H. / Leiserson, Ch. / Rivest, R.L. / Stein, C. 2009, 5.

[4] Since my topic is after all Mesopotamia and space is restricted, I shall not document these sweeping claims.

> One of the ways to help make computer science respectable is to show that it is deeply rooted in history, not just a short-lived phenomenon. Therefore it is natural to turn to the earliest surviving documents which deal with computation, and to study how people approached the subject nearly 4000 years ago.[5]

That Mesopotamian – by then known almost exclusively as Babylonian – mathematics concentrates on computation was (and is) undisputable. But Knuth wanted to show that Babylonian mathematics was built on *algorithms*. For this purpose, he used Neugebauer's translations, where everything is understood as purely numerical operations. He then concludes that

> The calculations described in Babylonian tablets are not merely the solutions to specific individual problems: they actually are general procedures for solving a whole class of problems. The numbers shown are merely included as an aid to exposition, in order to clarify the general method. [...] Thus the Babylonian procedures are genuine algorithms, and we can commend the Babylonians for developing a nice way to explain an algorithm by example as the algorithm itself was being defined.[6]

However, after going through a number of examples Knuth has to admit that

> So far we have seen only "straight-line" calculations, without any branching or decision-making involved. In order to construct algorithms that are really nontrivial from a computer scientist's point of view, we need to have some operations that affect the flow of control.
>
> But alas, there is very little evidence of this in the Babylonian texts.

That is, there is nothing corresponding to the above commands "UNTIL $i = n$" and "IF ..., ELSE ...". That is the reason that a single numerical example can be taken to correspond to an algorithm – if there is a choice or a limit, the example has to choose one possibility respectively to stop at the limit.

In order to get the taste of interesting algorithms, Knuth has to interpret a procedure involving a repetition *as if* it had contained a decision "UNTIL ..." – which it does not. So, all in all, Knuth may perhaps produce useful professional ideology, but his interpretation distorts what goes on in the texts, and cannot be considered serious historiography (nor was it probably meant to).

Also connected to modern concerns, but historiographically more to the point, was Wu Wenjun's work. He had worked himself as a pioneer on mechanized proof as a tool in creative mathematics[7] and saw that many procedures described in such Ancient Chinese mathematical texts as the *Nine*

[5] Knuth, D.E. 1972, 671.
[6] Knuth, D.E. 1972, 672–673.
[7] See Hudecek, J. 2014, 2 and passim.

Chapters on Arithmetic are, precisely, mechanical and thus algorithmic (somewhat more on this below). He is likely to have been inspired both by Knuth[8] and by the political conditions under which he lived, which called for legitimization of his metamathematical research with reference to Chinese tradition. However, he happened to have a much better case.[9] According to Hudecek,

> In comparison to Knuth's article, Wu's sole emphasis on Chinese mathematics appears narrow-minded. It might even be suggested that Wu tried to take away some credit from "rival" ancient civilizations in his later attempts to demonstrate the computational superiority of specific Chinese algorithms over Western ones.[10]

This, however, only shows that Hudecek has taken Knuth's claims at face value, without ever asking whether they were well-founded.

Closer inspection of Knuth's arguments suggests they are not (as already discussed). But insufficiency of arguments is no proof of falsity. So, in what follows I shall try to see whether algorithmic thinking *can* be traced in Mesopotamian mathematics, the insufficiency of Knuth's arguments notwithstanding.

First of all, as already said, Mesopotamian (and ancient Egyptian, and indeed all "scribal" or administrative) mathematics is overwhelmingly computational – in a phrase I have often used, its aim was "to find the right number". Mathematical riddles or school problems might well be "supra-utilitarian" – that is, look superficially as belonging to the kind of tasks scribes were supposed to deal with, even though they would never turn up in the real working practice of scribes and only had the merit to be more challenging than real-life problems. However, precisely because they had to look as if they were practical, even they were asking for "the right number", not for theory or deductive proof. Similarity *at this level* with the basic aim of computer-carried computation is thus not controversial at all. The crux is whether all such computation was performed by means of algorithms, as computation performed by machines has to be; and, as a next step, whether the whole computational endeavour can be characterized as "algorithmic" (what that means I shall elucidate in the final section).

So far, except when quoting I have mostly spoken of "Mesopotamian", not "Babylonian" mathematics, while my title refers to the latter category. The reason is that the only epochs that offer material allowing us to decide are the Old- and Neobabylonian (including Seleucid) periods. We may reasonably extrapolate our findings, but extrapolations are always to be taken with caution. So, I shall concentrate on "Babylonian" material.

---

8 See Hudecek, J. 2014, 118.

9 See Karine Chemla's copious work on (indubitable) algorithms in classical Chinese mathematics – e.g. Chemla, K. 1987; 1990; 1991.

10 Hudecek, J. 2014, 118.

The notion of "mathematics" calls for the final preliminary explication. Mesopotamian administration (as administrations generally) had been mathematical since proto-literate times. However, administrative documents contain the *results* of computations, not *their procedures.* So, the discussion has to build on what is known as "mathematical texts", that is, texts linked to the teaching of mathematics.

The two levels of Old Babylonian mathematics

When discussing Old Babylonian mathematics, we must distinguish two levels: firstly, that of numerical computation, where much – but we do not know exactly what – is likely to go back to the Ur III invention and implementation of the sexagesimal place value system; and secondly, problem culture, almost certainly new.

Numerical computation encompasses (1) addition and subtraction, (2) multiplication (as well known, the solution of division *questions* was solved via multiplication, namely by the reciprocal of the divisor, $a \div b = a \times \frac{1}{b}1$); and (3), the determination of the reciprocals of "regular" numbers (that is, numbers which possess a reciprocal that can be expressed as a finite sexagesimal fraction).[11]

Addition and subtraction were performed on some kind of counting board. According to calculational errors it must have had a structure in which a unit in one sexagesimal order of magnitude could be misplaced as a unit in a neighbouring order, but not as easily as 10 in the same or adjacent orders of magnitude.[12] We also know that it had four or five sexagesimal levels and was spoken of as "the hand" from ED III until Neo-Babylonian times.[13] No physical specimens have been found, but we may imagine *as possibilities* the two structures shown in the diagram (Fig. 1): either (above) separate cases for ones and tens or (below) cases for units as well as tens but distinct calculi for the two. In any case, subtraction was spoken of (in different Sumerian words, which excludes continuity at the level of language)

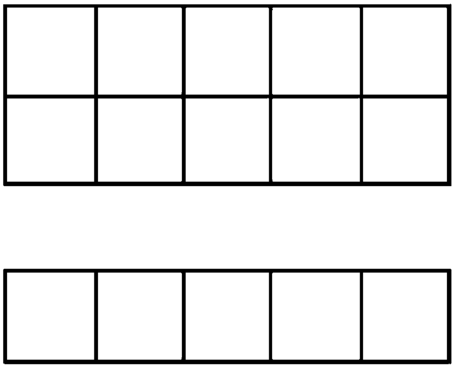

Fig. 1

[11] Strictly speaking even the finding of reciprocals of irregular numbers and the square roots of non-square numbers belong here – but the material at our disposal is insufficient for even a tentative discussion.

[12] Høyrup, J. 2002b.

[13] Proust, Ch. 2000, cf. Høyrup, J. 2002d; 2009.

during Ur III and in Seleucid times as "lifting up", which must refer the removal of counters.

Since our only sources for this are calculational errors and terminology, we have no evidence for how it was used in practice; however, some kind of fixed procedure – even with branchings analogous to those of our own algorithms for addition and subtraction – is likely: that is, a *non-trivial algorithm*, as Knuth would see it (not necessarily the same in all epochs and schools – we too have seen different algorithms for calculation with Arabic numerals).

For multiplication we do have some evidence: firstly, multiplication tables; secondly, tablets for "rough work", as they have been labelled by Eleanor Robson,[14] who was the first to single them out as a separate group.[15]

Multiplication tables are of two kinds. Firstly, there are tables which for a "principal number" a lists $1\times a$, $2\times a$, $3\times a$, ..., $19\times a$, $20\times a$, $30\times a$, $40\times a$, and $50\times a$. These, as well as the basic list of reciprocals, were copied repeatedly in the scribe school, for all we know with the purpose of being learned by heart. Apart from the "irregular" number 7, all principal numbers are either contained in the basic list of reciprocals, or easily obtained from it by halving or doubling. Combined multiplication tables were also produced.

If a was a familiar principal number and b one of the numbers 1, 2, 3, ..., 19, 20, 30, 40, and 50, ab could simply be remembered from a table. This hardly counts as an algorithm.

But not all products could simply be looked up, and that is where the tablets for rough work come in. The diagram contains an example.[16] There are no traces of intermediate calculations, but we may assume that the product has been found as the sum of partial products that were known from the multiplication tables, 5×5, 5×30, 5×7, 30×5, 30×30, 30×7, 7×5, 7×30, and 7×7 (this is also suggested by Christine Proust in her commentary) (Fig. 2). All of these would have to be put onto the counting board in the adequate order of magnitude.

7	35	
7	35	
57	30	25

Fig. 2

Could this calculation be categorized as an algorithm? Maybe. That depends on whether there was a fixed rule for the order of intermediate products (as here, "begin to the right of both factors, then move step by step toward the left in the lower factor, and when you have reached the left end, move one step to the left

[14] Robson, E. 1999, 7 and passim.

[15] Robson, E. ibid. generously refers to Jöran Friberg's notion (Friberg, J. 1990, 548) of "algorithm texts" as a precursor while pointing out that "it is unclear what he means by this". In any case, Friberg describes a much broader category than the one that is singled out by Robson.

[16] A transliteration of Ni 10246, made after the hand copy in Proust, Ch. 2008, 178.

in the upper factor, and repeat, ..."). That would correspond to the way we perform multiplications in our modern algorithm, digit by digit.

One consideration, however, speaks against the existence of a mechanical scheme: not all calculators would probably be equally familiar with all multiplication tables. We know that a table with principal number 50 existed, so some calculators would probably be able to insert a partial product 50×50 directly. But we also know that this table was not often copied – just one specimen is listed in MKT and MCT together, and part of the evidence for the use of a counting board involves a computation of 50×50 as (5×5)×(10×10). Instead of a mechanical rule, students were therefore probably taught to navigate according to their competence – that is, not according to an algorithm but by intelligent use of an open-ended rule.

The third branch of numerical computation, determination of reciprocals, encompasses several techniques. Firstly, of course, there is the determination of the reciprocals appearing in the standard table. That was probably done once and for all, as part of the general Šulgi reform after 2075 BCE, when not yet reciprocals but fractions $\frac{60}{n}$ were meant (which makes no difference in the floating-point sexagesimal system). At that initial moment methods will probably been *ad hoc* – before the advent of computer science, algorithms are the outcome of routine.

The production of new pairs of reciprocals from a known pair by successive doublings "to the left" and halvings "to the right"[17] seems to have been so much a routine that it can be characterized as an algorithm (evidently what Knuth would have seen as a trivial one, no choice being made). A more sophisticated method has been given the name "trailing-part algorithm" by Jöran Friberg;[18] it was first analysed by Abraham Sachs.[19] In order to show and discuss how it works we may take a simplified example, and pretend that A = 44.26.40[20] (the reciprocal of 1.21 = 81) does not appear in the standard table (it actually appears). We want to find its reciprocal $\frac{1}{A}$ from simpler pairs. We notice that the "trailing part" of the number is 6.40, which is the reciprocal of 9. We therefore write A as a sum, A = 44.20.0+0.6.40, and find that 9×A = 6.39.0+0.1.0 = 6.40. Now, 6.40 is still the reciprocal of 9, whence 9×9×A = 1. Therefore, $\frac{1}{A}$ = 9×9 = 81.

But if we had not taken notice of the last two sexagesimal digits but only of the last one, we might also have split A = 44.26.0+0.0.40. We recognize 40 as $^2/_3$, and therefore multiply by 3, getting 3A = 2.13.18+0.0.2 = 2.13.20. But the

[17] Friberg, J. 1990, 549.
[18] Friberg, J. 1990, 550.
[19] Sachs, A.J. 1947, 222–226.
[20] Numbers written with a point between the sexagesimal digits are floating point, that is, without determined absolute order of magnitude (but in sums, corresponding orders must be chosen). Final zeroes can therefore be left out, 6.40.0 = 6.40.

new trailing part 20 is $^1/_3$, and we therefore split $3A = 2.13.0{+}0.0.20$, and find that $3{\times}3A = 6.39.0{+}1.0 = 6.40$. Here the trailing part is 40, and we may therefore split $3{\times}3A = 6.0{+}0.40$, and multiply once more by 3, getting $3{\times}3{\times}3A = 18{+}2 = 20$. Now, as we have already pointed out, $20 = {}^1/_3$, and therefore $A = {}^1/_3{\times}{}^1/_3{\times}{}^1/_3{\times}{}^1/_3 = {}^1/_{81}$, and thus once again $\frac{1}{A} = 81$.

There is nothing in the procedure which automatically prescribes a particular choice for the splitting – once again, it seems to be up to the competence and remembered reciprocal pairs of the single reckoner. As observed by Proust [2012: 402, emphasis added] in a thorough discussion of the method based on real examples,

> All that is needed is to adjust for a suitable sequence. (In the case of 2.13.20, we may take 20, or 3.20, or even 13.20.) [...] *In the majority of cases*, the scribe chose, from among the possible factors, the "largest" (3.20 rather than 20), in order to render the algorithm faster.[21]

Once more, we seem to be confronted, not with an algorithm but with intelligent use of an open-ended rule.

This concerned the level of numerical computation. Even though our evidence is Old Babylonian, the procedures for using the place value system are likely to go back to Ur III (with the possible exception of the trailing part algorithm, which is no necessary constituent of the system). The use of the reckoning board is likely to have remained more or less the same at least since ED III – but regarding the precise procedures used on this board as well as possible changes occasioned by the coupling to the place-value system during Ur III we are left in the dark.

We shall now turn to the level of problem culture. Simple mathematical problems, utilitarian as well as supra-utilitarian, had been used in the mathematical training of futures scribes since ED III, but that practice seems to have been interrupted during Ur III.[22] In any case, the flourishing of often complex and very often supra-utilitarian problems in the Old Babylonian school (probably beginning in 19th-century BCE Ur but unfolding in early 18th-century BCE Eshnunna[23] is a significant innovation reflecting the new scribal culture of the period.

Problems were exactly what inspired Knuth. Relying on the translations at his disposal, he read the procedures as sequences of purely numerical prescriptions. A better understanding of the terminology shows that many of them – those that are often taken to represent "Babylonian second-degree algebra" – instead prescribe geometric manipulations.[24] They might still be algorithms in spite of that, unless

---

[21] Proust, Ch. 2010, 402, emphasis added.
[22] See Høyrup, J. 2002c.
[23] Bronze Age dates according to the middle chronology.
[24] See, for example, Høyrup, J. 2002a.

we take the above expression "computational procedure" to mean that it has to regard pure numbers, not measures of distances and areas. Even the prescription of how to construct an equilateral triangle in Euclid's *Elements* I.1 can very well be understood as an algorithm:[25]

> *On a given finite straight line to construct an equilateral triangle*
> Let AB be the given finite straight line. Thus it is required to construct an equilateral triangle on the straight line AB. With centre A and distance AB let the circle BCD be described; again, with centre B and distance BA let the circle ACE be described; and from the point C, in which the circles cut one another, to the points A, B let the straight lines CA, CB be joined.[26]

A proof of the correctness of the construction follows, but computer algorithms also often contain non-executable explanations in "comment" fields. The procedure is certainly not "computational" in the usual numerical sense, and the input- and output-"values" are not numbers but given points and segments. Unless we are very pedantic, however, this *is* an (unbranched) algorithm.

So, is it reasonable to describe the Old Babylonian solutions to "quadratic equations" as algorithms? We may look at the very simplest mixed second-degree problem, BM 13901 #1:[27]

1. The surface and my confrontation I have accumulated: 45′ is it. 1, the projection,
2. you posit. The moiety of 1 you break, 30′ and 30′ you make hold.
3. 15′ to 45′ you append: by 1, 1 is equal. 30′ which you have made hold
4. in the inside of 1 you tear out: 30′ the confrontation.

---

[25] I omit the diagram, just as the Old Babylonian clay tablets omit them; in both cases they are easily reconstructed from the words, once the terminology is understood.

[26] Ed. trans. Heath, Th.L. 1926, I, 241.

[27] With one minor modification I follow the translation in Høyrup, J. 2002a, 50. A "confrontation" is a configuration characterized by the confrontation of equals, that is, a square frame parametrized by ("being") its side and *having* an area (while our square basically *is* the area and *has* a side). The "projection" is that length 1 which, when applied to a segment c as width, produces a rectangle with area c. To "make a and b hold" stands for constructing a rectangle contained ("held") by the sides a and b. That s "is equal" by A means that s is the side of the area A laid out as a square. The rest can be followed in the diagram.

The text was first published by F. Thureau-Dangin in 1936.

The statement explains (in our terms) that the sum of the area c^2 and a side c of a square is $45' = {}^3/_4$. Therefore, the side it provided with a width (the "projection") 1, which produces a rectangle with area $1 \times c = c$. This rectangle is bisected and its outer half moved around, which gives us a gnomon with a missing square of area $30' \times 30' = 15' \times (({}^1/_2)^2 = {}^1/_4)$. Adding this to the gnomon we get a square with area 1, and therefore also side 1. Removing the part that was moved around we are left with the original side c, which must therefore be $1–30' = 30' = {}^1/_2$.

Next follow a number of other problems about a single square (for brevity in modern transcription):

(2) $c^2 – c = 14`30$

(3) $\left(1-\frac{1}{3}\right) c^2 + \frac{1}{3} c = 20'$

(4) $\left(1-\frac{1}{3}\right) c^2 + c = 4`46°40'$

(5) $c^2 + (1+\frac{1}{3})c = 55'$

(6) $c^2 + \frac{2}{3} c = 35'$

(7) $11c^2 + 7c = = 6°15'$

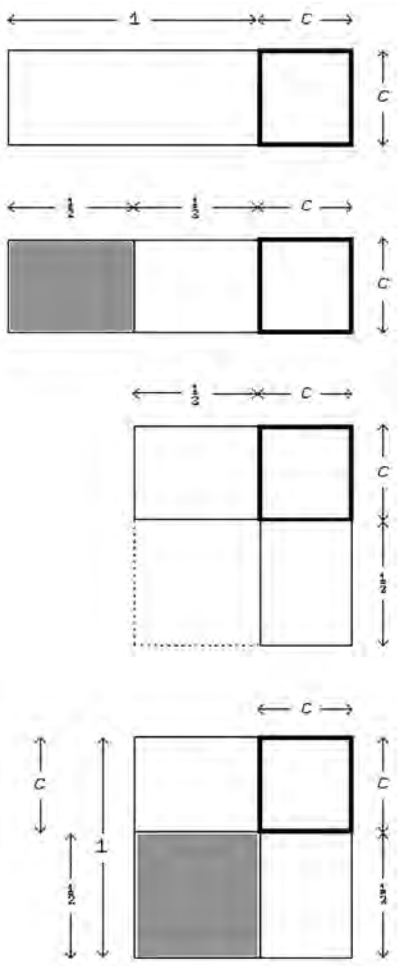

Fig. 3

(2) obviously cannot be solved exactly like the first problem, but the procedure is as close as possible. All those with a structure $ac^2 + bc = k$ are transformed via a multiplication into $(ac)^2 + b(ac) = ka$ $(ac)^2$, and after that they follow the procedure of the first problem step by step. So far it seems reasonable to see this as an instance of an algorithm (without branching, evidently).

However, a look at problem (14) of the same tablet is informative. It deals with two squares. We may designate their sides c_1 and c_2. It is stated that

$$c_1{}^2 + c_2{}^2 = 25'25''$$
$$c_2 = \frac{2}{3} c_1 + 5'$$

Here, a new square side c is introduced, $c_1 = 1 \times c$, $c_2 = \frac{2}{3} c + 5'$, and it is found (geometrically, but for brevity once more in modern translation) that

$$1°\,26'40'' c^2 + \mathrm{p}c = 25'$$

which as in problems (3)–(7) is transformed into

$$(1°\ 26'40''c)^2+p\times(1°\ 26'40''c)=1°\ 26'40''\times25'$$

p would have to be calculated as $2\times5'\times40'$, since $(\frac{2}{3}c+5')^2 = (40'c+5')^2 = 26'40''c^2+2\times5'\times40'+25''$.

However, the author of the text only finds $5'\times40' = 3'20''$, that is, $2p$. The reason is that he *does not treat the procedure of problem (1) as an algorithm* that can be used as a subroutine within more complicated procedures. He knows, indeed, that $2p$, the number of sides, will have to be halved; therefore, instead of first doubling $3'20''$ and then halving the outcome slightly afterwards, he omits both steps. Once again, what we see is not an algorithm but flexible use of an open-ended rule. That problems (1) and (3)–(7) look as if they followed an algorithm is simply due to the fact that there is no need to make intelligent use of the standard procedure.[28]

Another famous text, AO 8862[29] presents a sequence of three rectangle problems that *could* all be reduced to the same standard configuration. Instead the text teaches three *different* ways to do it. Similarly, the text BM 85200 + VAT 6599[30] teaches one trick to overcome the problem arising from the use of different metrologies for horizontal and for vertical distances when the length of a rectangular prismatic excavation is given, leaving depth and width as unknowns (to turn the prism around mentally), and another one when width is given, leaving the depth and the length as unknowns (use of a conversion factor).

These two texts clearly aim at training the creativity of those who are taught (or perhaps displaying the creativity of the author), not just at training mechanical algorithms. What they offer (and what all the other problem texts explaining a procedure offer) are not algorisms but *paradigmatic examples*, examples to be emulated with as much variation and individual fantasy as required.

An interesting supplement is offered by the twin texts VAT 8389+VAT 8391.[31] They contain a number of problems about two fields, for which the sum of or difference between the areas is given together with the sum of or difference between the rents paid (sometimes, simpler, just one of the entities instead of sum or difference). The problems are thus of the first degree, and it would not be too difficult to make use of a standard procedure reducing them to a shared algorithm. But that is not done, the methods are individual and fitted heuristically to

[28] According to an anecdote, a mathematician is taught how to make tea if he finds an empty kettle on the gas stove. He first has to fill it with water, etc. Asked then what to do if the kettle is already filled, he suggests first to empty it, thereby reducing the situation to the one already known – that is, he takes what he is taught as an algorithm. Out Babylonian author is obviously not of the kind.

[29] MKT I, 108–117, cf. Høyrup, J. 2002a: 162–173; probably from Larsa and to be dated around 1750 BCE.

[30] MKT I, 193–208, cf. Høyrup, J. 2002a: 137–162; probably from Sippar and to be dated to the late 17th century BCE.

[31] MKT I, 317–335, cf. Høyrup, J. 2002a, 77–85.

the single situations. However, the givens, stated in the units of practical agriculture (BÙR and GUR), have to be converted into the "basic units" SAR and SILÀ serving in the place value system. These conversions, when they cannot be read out from a metrological table, are made in meticulous detail, as if rendering a mechanical procedure trained *ad nauseam* – an algorithm. This may correspond to the training of rank-and-file scribes (for whom the sophisticated problems were hardly meant):[32] for certain, not too difficult tasks (for example, multiplications) they may have been taught to use their brains and navigate according to what this brain kept in memory; for others (for example, the use of metrological tables, which really had to be memorized), they had to follow strict algorithmic schemes.

Later Periods

We have a few traces of mathematics but no "mathematical texts" from the millennium following upon the collapse of the Old Babylonian social system and school. The few surviving mathematical texts from the fifth century BCE[33] do not allow to draw any conclusions as to whether their prescriptions were still understood as paradigmatic examples or as representations of algorithms. The same may be said about the few Seleucid mathematical texts we possess.[34] However, the density of mathematical activity of the kind reflected in these texts appears to have been so low that nothing would have called for an algorithmic mechanization.

"of the kind reflected in these texts" – but there was another kind of mathematical activity from the Achaemenid to the Seleucid and even the Arsacid period, the one involved in mathematical astronomy. This activity was intense, at least for the small group of participants, and here mechanization of the calculations will have been quite meaningful. We also have direct evidence for this. The text BM 22282+42294, probably of Achaemenid date, which was published by Lis Brack-Bernsen and Hermann Hunger in 2008, teaches how to find out whether the month is hollow (29 days) or full (30 days). The prescriptions are in fully algorithmic style, even (as asked for by the topic) containing explicit IF commands. Such must also have been present in the algorithms producing the zigzag-functions of later mathematical astronomy, deciding when and exactly

[32] It should be emphasized that we have no direct evidence as concerns the intended learners of the Old Babylonian sophisticated problem texts. The format, and observations as this one, show that the endeavour was somehow connected to the school – Old Babylonian sophisticated mathematics grew from school soil. But that is all, and a format may simply reflect expectations to how mathematics ought to be formulated (as nowadays it has to be formulated in the second personal plural, "we" see, do, etc.).

[33] W 23 191-x (Friberg, J. / Hunger, H. / al-Rawi, F.N.H. 1990); W 23 291 (Friberg, J. 1997); dating according to Robson, E. 2008, 227–230.

[34] BM 34568 (MKT III, 14–19), AO 6484 (MKT I, 96–102), VAT 7848 (MCT, 141*f*).

how to turn from zig to zag. So, if Knuth had looked at astronomy, and if he had had access to material that was only published half a century after he wrote, he would have had a much better (and, in his opinion, much more interesting) case.

Summing up

All in all, we find some algorithms in Babylonian mathematics, even though it was mostly taught by means of paradigmatic examples giving space to flexibility when flexibility was needed. Only the mathematics of Late Babylonian astronomy was apparently based on algorithms, in which the limited flexibility that was required was built into the algorithms by means of IF commands.

But the question of my title is whether Babylonian mathematics was *algorithmic*. What do I mean by that?

I shall explain by means of a paradigmatic example: classical Chinese mathematics.[35] In the *Nine Chapters on Arithmetic*[36] from the Han period we find a general structure where an abstract rule is set out first, as a counting rod algorithm; then concrete examples follow, which vary the numerical parameters and the real-life topic but have the same mathematical structure. For example, chapter III claims to deal with distribution weighted according to rank, actually the topic of the first example; the second problem deals with payment according to consumption, the third with customs paid according to possession. These follow the same computational scheme, and we may adequately speak of an algorithmic organization (a straight-line algorithm, for sure). The fourth problem, however, goes beyond the initial algorithm. A woman weaves each day twice as much as the day before, and in five days she has woven for 5 *chi*. The weights are given immediately as 1, 2, 4, 8 and 16, respectively: only that part of the calculation that fits inside the algorithm is specified, for what falls outside the algorithmic part the outcome is just stated. The *stylistic ideal* is thus algorithmic, centred on procedures, even when the actual task goes beyond the algorithmic framework. Most of the texts on which examinations were based are similar.[37] Commentaries (for instance, that of Liu Hui) explain why algorithms work, and by being commentaries to the algorithms confirm the central position of these. Commentaries also speak about constructing new procedures or algorithms, not only about using procedures flexibly in non-standard situations;[38] even when it comes to mathematical activity, the algorithm was thus central.

In that sense we may speak of classical Chinese mathematical culture as algorithmic. And in that same sense we may conclude that Babylonian mathematical culture, probably excepting the branch that served mathematical astro-

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35 For further elaboration, see Høyrup, J. 2015.
36 Ed. trans. Chemla, K. / Guo, Sh. 2004.
37 Siu, M.-K. / Volkov, A. 1999, 93.
38 Siu, M.-K. / Volkov, A. 1999, 94.

nomy, was *not* algorithmic. Use of algorithms was not central but subordinate, and algorithms did not constitute a stylistic ideal. Babylonian mathematics was certainly *computational*, but that is a different matter – only in recent decades has "computational" become a quasi-synonym of "algorithmic", and Acton's textbook from 1970 presupposes that students program their solutions in FORTRAN, PL/1 or ALGOL and have access to a computer.[39]

Abbreviations

MCT Neugebauer, O. / Sachs, A. 1945.

MKT Neugebauer, O. 1935–1937.

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[39] Acton, F.S. 1990, xvii.

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